# #1 Make sense of problems and persevere in solving them.

| Summary of Standards for Mathematical Practice | Questions to Develop Mathematical Thinking |
|---|---|
| **1. Make sense of problems and persevere in solving them.**<br>• Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem.<br>• Plan a solution pathway instead of jumping to a solution.<br>• Monitor the progress and change the approach if necessary.<br>• See relationships between various representations.<br>• Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.<br>• Students ask themselves, "Does this make sense?" and understand various approaches to solutions. | How would you describe the problem in your own words?<br>How would you describe what you are trying to find?<br>What do you notice about...?<br>What information is given in the problem?<br>Describe the relationship between the quantities.<br>Describe what you have already tried. What might you change?<br>Talk me through the steps you've used to this point.<br>What steps in the process are you most confident about?<br>What are some other strategies you might try?<br>What are some other problems that are similar to this one?<br>How might you use one of your previous problems to help you begin?<br>How else might you organize...represent...show...? |

## Implementation Characteristics: What does it look like in planning and delivery?

Task: elements to keep in mind when determining learning experiences  Teacher: actions that further the development of math practices within their students

**Task:**

- [ ] Requires students to engage with conceptual ideas that underlie the procedures to complete the task and develop understanding.
- [ ] Requires cognitive effort - while procedures may be followed, the approach or pathway is not explicitly suggested by the task, or task instructions and multiple entry points are available. The problem focuses students' attention on a mathematical idea and provides an opportunity to develop and/or use mathematical habits of mind.
- [ ] Allows for multiple entry points and solution paths as well as, multiple representations, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations to develop meaning.
- [ ] Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- [ ] Requires students to defend and justify their solutions.

**Teacher:**

- [ ] Allows students time to initiate a plan; uses question prompts as needed to assist students in developing a pathway.
- [ ] Continually asks students if their plans and solutions make sense.
- [ ] Questions students to see connections to previous solution attempts and/or tasks to make sense of current problem.
- [ ] Consistently asks to defend and justify their solution by comparing solution paths.
- [ ] Differentiates to keep advanced students challenged during work time

# #2 Reason abstractly and quantitatively.

| Summary of Standards for Mathematical Practice | Questions to Develop Mathematical Thinking |
|---|---|
| **2. Reason abstractly and quantitatively.**<br>• Make sense of quantities and their relationships.<br>• Able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships.<br>• Understand the meaning of quantities and are flexible in the use of operations and their properties.<br>• Create a logical representation of the problem.<br>• Attend to the meaning of quantities, not just how to compute them. | What do the numbers used in the problem represent?<br>What is the relationship of the quantities?<br>How is _____ related to _____?<br>What is the relationship between _____and _____?<br>What does_____mean to you? (e.g. symbol, quantity, diagram)<br>What properties might we use to find a solution?<br>How did you decide in this task that you needed to use...? Could you have used another operation or property to solve this task? Why or why not? |

## Implementation Characteristics: What does it look like in planning and delivery?

Task: elements to keep in mind when determining learning experiences  Teacher: actions that further the development of math practices within their students

**Task:**

- ☐ Includes questions that require students to attend to the meaning of quantities and their relationships, not just how to compute them.
- ☐ Consistently expects students to convert situations into symbols in order to solve the problem and then requires students to explain the solution within a meaningful situation.
- ☐ Contains relevant, realistic content.

**Teacher:**

- ☐ Expects students to interpret, model, and connect multiple representations.
- ☐ Asks students to explain the meaning of the symbols in the problem and in their solution.
- ☐ Expects students to give meaning to all quantities in the task.
- ☐ Questions students so that understanding of the relationships between the quantities and/or the symbols in the problem and the solution are fully understood.

# #3 Construct viable arguments and critique the reasoning of others.

| Summary of Standards for Mathematical Practice | Questions to Develop Mathematical Thinking |
|---|---|
| **3. Construct viable arguments and critique the reasoning of others.**<br>• Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments.<br>• Justify conclusions with mathematical ideas.<br>• Listen to the arguments of others and ask useful questions to determine if an argument makes sense.<br>• Ask clarifying questions or suggest ideas to improve/revise the argument.<br>• Compare two arguments and determine correct or flawed logic. | What mathematical evidence supports your solution?<br>How can you be sure that...? / How could you prove that...? Will it still work if...?<br>What were you considering when...?<br>How did you decide to try that strategy?<br>How did you test whether your approach worked?<br>How did you decide what the problem was asking you to find? (What was unknown?)<br>Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not?<br>What is the same and what is different about...?<br>How could you demonstrate a counter-example? |

## Implementation Characteristics: What does it look like in planning and delivery?

Task: elements to keep in mind when determining learning experiences  Teacher: actions that further the development of math practices within their students

**Task:**

- ☐ Is structured to bring out multiple representations, approaches, or error analysis.
- ☐ Embeds discussion and communication of reasoning and justification with others.
- ☐ Requires students to provide evidence to explain their thinking beyond merely using computational skills to find a solution.
- ☐ Expects students to give feedback and ask questions of others' solutions.

**Teacher:**

- ☐ Encourages students to use proven mathematical understandings, (definitions, properties, conventions, theorems, etc.), to support their reasoning.
- ☐ Questions students so they can tell the difference between assumptions and logical conjectures.
- ☐ Asks questions that require students to justify their solution and their solution pathway.
- ☐ Prompts students to respectfully evaluate peer arguments when solutions are shared.
- ☐ Asks students to compare and contrast various solution methods.
- ☐ Creates various instructional opportunities for students to engage in mathematical discussions (whole group, small group, partners, etc.).

# #4 Model with mathematics.

| Summary of Standards for Mathematical Practice | Questions to Develop Mathematical Thinking |
|---|---|
| **4. Model with mathematics.**<br>• Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize).<br>• Apply the math students know to solve problems in everyday life.<br>• Able to simplify a complex problem and identify important quantities to look at relationships.<br>• Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation.<br>• Reflect on whether the results make sense, possibly improving/revising the model.<br>• Ask themselves, "How can I represent this mathematically?" | What number model could you construct to represent the problem?<br>What are some ways to represent the quantities?<br>What's an equation or expression that matches the diagram? number line? chart? table?<br>Where did you see one of the quantities in the task in your equation or expression?<br>Would it help to create a diagram, graph, table, ...?<br>What are some ways to visually represent...?<br>What formula might apply in this situation? |

## Implementation Characteristics: What does it look like in planning and delivery?

Task: elements to keep in mind when determining learning experiences  Teacher: actions that further the development of math practices within their students

**Task:**

- [ ] Is structured so that students represent the problem situation and their solution symbolically, graphically, and/or pictorially (may include technological tools) appropriate to the context of the problem.
- [ ] Invites students to create a context (real-world situation) that explains numerical/symbolic representations.
- [ ] Asks students to take complex mathematics and make it simpler by creating a model that will represent the relationship between the quantities.
- [ ] Requires students to identify variables, compute and interpret results, report findings, and justify the reasonableness of their results and procedures within context of the task.

**Teacher:**

- [ ] Demonstrates and provides student's experiences with the use of various mathematical models.
- [ ] Questions students to justify their choice of model and the thinking behind the model.
- [ ] Asks students about the appropriateness of the model chosen.
- [ ] Assists students in seeing and making connections among models.
- [ ] Give students opportunity to evaluate the appropriateness of the model.

# #5 Use appropriate tools strategically.

| Summary of Standards for Mathematical Practice | Questions to Develop Mathematical Thinking |
|---|---|
| **5. Use appropriate tools strategically.**<br>• Use available tools recognizing the strengths and limitations of each.<br>• Use estimation and other mathematical knowledge to detect possible errors.<br>• Identify relevant external mathematical resources to pose and solve problems.<br>• Use technological tools to deepen their understanding of mathematics.<br>• Use mathematical models for visualizing and analyzing information | What mathematical tools could we use to visualize and represent the situation?<br>What information do you have?<br>What do you know that is not stated in the problem?<br>What approach are you considering trying first?<br>What estimate did you make for the solution?<br>In this situation would it be helpful to use.a graph..., number line..., ruler..., diagram..., calculator..., manipulative?<br>Why was it helpful to use ____?<br>What can using a _____ show us that _____may not?<br>In what situations might it be more informative or helpful to use...? |

## Implementation Characteristics: What does it look like in planning and delivery?

Task: elements to keep in mind when determining learning experiences  Teacher: actions that further the development of math practices within their students

**Task:**

- ☐ Requires multiple learning tools. (Tools may include: manipulatives (concrete models), calculator, measurement tools, graphs, diagrams, spreadsheets, statistical software, etc.)
- ☐ Requires students to determine and use appropriate tools to solve problems.
- ☐ Requires students to demonstrate fluency in mental computations.
- ☐ Asks students to estimate in a variety of situations:
  - -a task when there is no need to have an exact answer
  - -a task when there is not enough information to get an exact answer
  - -a task to check if the answer from a calculation is reasonable

**Teacher:**

- ☐ Demonstrates and provides students experiences with the use of various math tools. A variety of tools are within the classroom learning environment and readily available.
- ☐ Allows students to choose appropriate learning tools and questions students as to why they chose the tools they used to solve the problem.
- ☐ Consistently models how and when to estimate effectively, and requires students to use estimation strategies in a variety of situations.
- ☐ Asks student to explain their mathematical thinking with the chosen tool.
- ☐ Asks students to explore other options when some tools are not available.

# #6 Attend to precision.

| Summary of Standards for Mathematical Practice | Questions to Develop Mathematical Thinking |
|---|---|
| **6. Attend to precision.**<br>• Communicate precisely with others and try to use clear mathematical language when discussing their reasoning.<br>• Understand meanings of symbols used in mathematics and can label quantities appropriately.<br>• Express numerical answers with a degree of precision appropriate for the problem context.<br>• Calculate efficiently and accurately. | What mathematical terms apply in this situation?<br>How did you know your solution was reasonable?<br>Explain how you might show that your solution answers the problem.<br>Is there a more efficient strategy?<br>How are you showing the meaning of the quantities?<br>What symbols or mathematical notations are important in this problem?<br>What mathematical language..., definitions..., properties can you use to explain...?<br>How could you test your solution to see if it answers the problem? |

## Implementation Characteristics: What does it look like in planning and delivery?

Task: elements to keep in mind when determining learning experiences  Teacher: actions that further the development of math practices within their students

**Task:**

- [ ] Requires students to use precise vocabulary (in written and verbal responses) when communicating mathematical ideas.
- [ ] Expects students to use symbols appropriately.
- [ ] Embeds expectations of how precise the solution needs to be (some may more appropriately be estimates).

**Teacher:**

- [ ] Consistently demands and models precision in communication and in mathematical solutions. (*uses and models correct content terminology*).
- [ ] Expects students to use precise mathematical vocabulary during mathematical conversations. (*identifies incomplete responses and asks students to revise their response*).
- [ ] Questions students to identify symbols, quantities, and units in a clear manner

# Implementing Standards for Mathematical Practices

**#7 Look for and make use of structure.**

| Summary of Standards for Mathematical Practice | Questions to Develop Mathematical Thinking |
|---|---|
| **7. Look for and make use of structure.**<br>• Apply general mathematical rules to specific situations.<br>• Look for the overall structure and patterns in mathematics.<br>• See complicated things as single objects or as being composed of several objects. | What observations do you make about...?<br>What do you notice when...?<br>What parts of the problem might you eliminate? simplify?<br>What patterns do you find in...?<br>How do you know if something is a pattern?<br>What ideas have we learned before that were useful in solving this problem?<br>What are some other problems that are similar to this one?<br>How does this relate to...?<br>In what ways does this problem connect to other mathematical concepts? |

**Implementation Characteristics: What does it look like in planning and delivery?**

Task: elements to keep in mind when determining learning experiences  Teacher: actions that further the development of math practices within their students

**Task:**

☐ Requires students to look for the structure within mathematics in order to solve the problem. (i.e. – decomposing numbers by place value, working with properties, etc.).

☐ Asks students to take a complex idea and then identify and use the component parts to solve problems. i.e. building on the structure of equal sharing, students connect their understanding to the traditional division algorithm. When "unit size" cannot be equally distributed, it is necessary to break down into a smaller "unit size". (example below)

```
4 )351
  -32
   31
  -28
    3
```

3 *hundreds* units cannot be distributed into 4 equal groups. Therefore, they must be broken down into *tens* units.

There are now 35 *tens* units to distribute into 4 groups. Each group gets 8 sets of tens, leaving 3 extra *tens* units that need to become *ones* units.

This leaves 31 *ones* units to distribute into 4 groups. Each group gets 7 *ones* units, with 3 *ones* units remaining. The quotient means that each group has 87 with 3 left.

☐ Expects students to look at problems and think about them in an unconventional way that demonstrates a deeper understanding of the mathematical structure---leading to a more efficient way of solving the problem. They recognize and identify structures from previous experience(s) and apply this understanding in a new situation. (i.e. 7 x 8 = (7 x 5) + (7 x 3) OR 7 x 8 = (7 x 4) + (7 x 4). New situations could be distributive property, area of composite figures, multiplication fact strategies.)

**Teacher:**

☐ Encourages students to look at or something they recognize and have students apply the information in identifying solution paths (i.e. composing/decomposing numbers and geometric figures, identifying properties, operations, etc.).

☐ Expects students to explain the overall structure of the problem and the *big math idea* used to solve the problem.

*Secondary Example end of document*

# #8 Look for and express regularity in repeated reasoning.

| Summary of Standards for Mathematical Practice | Questions to Develop Mathematical Thinking |
|---|---|
| **8. Look for and express regularity in repeated reasoning.**<br>• See repeated calculations and look for generalizations and shortcuts.<br>• See the overall process of the problem and still attend to the details.<br>• Understand the broader application of patterns and see the structure in similar situations.<br>• Continually evaluate the reasonableness of their intermediate results. | Will the same strategy work in other situations?<br>Is this always true, sometimes true or never true?<br>How would you prove that...?<br>What do you notice about...?<br>What is happening in this situation?<br>What would happen if...?<br>Is there a mathematical rule for...?<br>What predictions or generalizations can this pattern support?<br>What mathematical consistencies do you notice? |

## Implementation Characteristics: What does it look like in planning and delivery?

Task: elements to keep in mind when determining learning experiences  Teacher: actions that further the development of math practices within their students

**Task:**

- [ ] Addresses and connects to prior knowledge in a non-routine way.
- [ ] Present several opportunities to reveal patterns or repetition in thinking so generalizations can be made.
- [ ] Requires students to see patterns or relationships in order to develop a mathematical rule.
- [ ] Expects students to discover the underlying structure of the problem and come to a generalization.
- [ ] Connects to a previous task to extend learning of a mathematical concept.

**Teacher:**

- [ ] Encourages students to connect task to prior concepts and tasks.
- [ ] Prompts students to generate exploratory questions based on current tasks.
- [ ] Asks what math relationships or patterns can be used to assist in making sense of the problem.
- [ ] Asks for predictions about solutions at midpoints throughout the solution process and encourages students to monitor each other's intermediate results.
- [ ] Questions students to assist them in creating generalizations based on repetition in thinking and procedures.

**# 7 Look for and make use of structure**

**Secondary Examples:**

What does it mean to look for and make use of structure?

- Students can look at problems and think about them in an unconventional way that demonstrates a deeper understanding of the mathematical structure – leading to a more efficient means to solving the problem.

Example problem:

- Solve for $x$ : $3(x-2) = 9$

Rather than approach the problem above by distributing or dividing, a student who uses structure would identify that the equation is saying 3 times something is 9 and thus the quantity in parenthesis must be 3.

Example problem:

- Solve for $x$ : $\frac{3}{x-1} = \frac{6}{x+3}$

The "typical" approach to the above problem would be to cross multiply and solve; a student who identifies and makes use of structure sees that the left side can be multiplied by 2 to create equivalent numerators... then simply set the denominators equal and solve.